On Mobile Robot Localization From Landmark Bearings

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Abstract

The paper deals with the problem of robot localization from noisy landmark bearings measured by the robot. We present a new localization method which is based on linear constraints, one due to each bearing measurement. This linear system can be solved at low computational cost but yields not very accurate results. We therefore transform the system to an equivalent linear system which yields virtually optimal results at a small fraction of the cost of a non-linear optimization method which usually achieves the optimal result. Experimental results showing the quality of the results and the low computational cost are presented.

1 Introduction

When mobile robots or Automated Guided Vehicles (AGV's) are moving in their workspaces one of the basic problems which needs to be solved is for the robot to know its position and orientation in the plane as accurately as possible. One of the standard methods of performing this task is to put landmarks in known locations in the workspace. In any place in the workspace the robot is able to measure the bearings to a sufficient number of these landmarks. Using three or more such measurements the robot is able to estimate its position and orientation in the plane [19, 17, 10, 5].

The most widely used method for computing this estimate is a geometric method based on the idea that the angle between two such bearing measurements yields the constraint that the robot's position is limited to lie on a circle. Adding an additional bearing measurement yields two more circles whose intersection is the desired location. The problem begins when there are errors in the measurements. To overcome this problem more than three landmarks are placed in the workspace and all the measured bearings have to be used to get the optimal estimate for the robot's position. The geometric method does not lend itself naturally to more than three bearing measurements. Thus costly non-linear minimization techniques with their known problems have to be employed.

We would like therefore to take a different approach which has been presented in [4] but without dealing with the accuracy problems. We will use each bearing measured to define a constraint on the position and orientation of the robot. These will be linear constraints. When given three or more bearing measurements we have that number of constraints and using numerical linear algebra techniques (Singular Value Decomposition) we are able to solve this linear system and find the best solution for this system very efficiently.

The problem is however, that when there is uncertainty in the bearing measurements, the solution to this linear system is not the optimal solution to the original problem. We therefore present several transformations of the linear system yielding more accurate results. All these transformations have negligible computational cost but improve the accuracy of the results considerably. In the final algorithm we present a simple optimization procedure whose computational cost is twice or three times more than the previous algorithm which further improves the accuracy. The quality of the results are essentially equal to the optimal results which can be achieved by non-linear optimization but at a much higher cost.

For many applications it is very important not only to estimate the position and orientation of the robot but also to estimate the accuracy of the estimate by estimating its covariance matrix. Without these estimates it is impossible to guarantee robustness of motion planning algorithms which use the localization algorithm. The covariance matrix is estimated by simply conducting several dozen experiments on input with synthetically added noise. This method is different then the method presented in [8] which computes a worst case estimate of the uncertainty region when bounded error is assumed. We compute the estimated accuracy over all locations in several examples of workspaces.

This can then be used to plan robust safe and fast motions plans for the robot as shown in [7, 18, 13, 15, 6, 2, 3] and to plan optimally the placement of the landmarks in the environment when given the accuracy requirements on the robot's localization at certain positions in the workspace [16].

The paper continuous as follows: In Section 2 we review the geometric method for estimating the robot's position and orientation and show also the algebraic method. In Section 3 we describe the requirements needed to produce more accurate results and present transformations to the derived constraints which improve the accuracy of the results. These algorithms are then run and experimental results are presented. In Section 4 we conclude the paper by discussing applications for this algorithm and further research directions.

2 The Basic Methods

2.1 Problem Statement

We are given a set of n landmarks whose positions $\mathbf{P}_i \in \mathbb{R}^2$ are known. The robot is at an unknown position $\mathbf{P}$ and orientation θ. The bearing measurements by the robot to these landmarks are β_i.

The problem is illustrated in Figure 1. Here three 2D points $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$ are viewed by the robot in directions $\beta_1, \beta_2, \beta_3$ respectively. The goal is to find the position $\mathbf{P}$ and orientation θ of the robot.

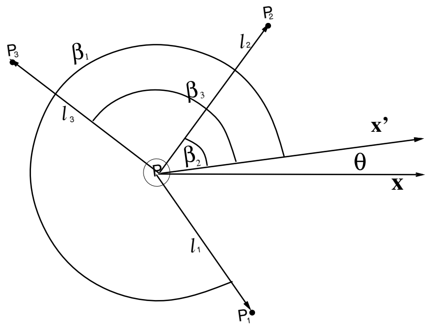

Figure 1: An illustration of a planar mobile robot which detects three known landmarks in directions $\beta_1, \beta_2, \beta_3$. The goal is to find the robot's position and orientation in the plane.

2.2 The Geometric Method

The geometric method is based on the following constraint. When given two landmarks in positions $\mathbf{P}_i$ and $\mathbf{P}_j$ and whose measured bearings are β_i and β_j there exists a circle going through $\mathbf{P}_i$ and $\mathbf{P}_j$ such that the angle difference between the directions of the two bearings is $\beta_i - \beta_j$ (See Fig 2). When given three such landmarks, the intersection of the three circles which are due to the three pairs of measurements, yield the unique solution to the problem. There exists an inherent ambiguous situation when the robot's position is co-circular with the positions of the three landmarks. In this case the robot's position can only be estimated to lie anywhere on the circle on which the landmarks lie and additional landmark bearing measurements are needed to disambiguate the robot's position.

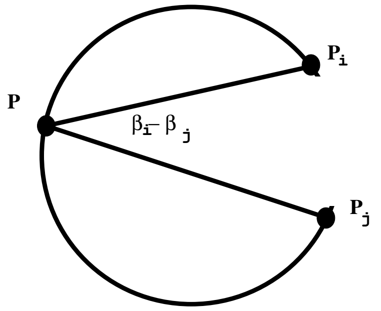

Figure 2: Given two landmarks P_i and P_j whose bearing measurements are β_i and β_j respectively. The possible robot positions lie on a circle illustrated in the figure.

2.3 The Algebraic Method

In the method we will present here we look at the problem as a coordinate transformation problem from the global coordinates $\mathbf{P}_i$ to the robot coordinates $\mathbf{M}_i$. The change is due to the robot's position $\mathbf{P}$ and orientation θ. Applying the transformation to a single bearing measurement yields the following equation.

$$\mathbf{M}_i = l_i(\cos\beta_i, \sin\beta_i) = R(\mathbf{P}_i - \mathbf{P}) = R\mathbf{P}_i + \mathbf{T} \quad (1)$$

where R is a planar rotation matrix by θ, $\mathbf{T} = -R\mathbf{P}$ and l_i is the unknown distance from $\mathbf{P}$ to $\mathbf{P}_i$. This can be written as the following two equations:

$$\begin{aligned} l_i \cos\beta_i &= (\cos\theta, \sin\theta) \cdot \mathbf{P}_i + \mathbf{T}_x \\ l_i \sin\beta_i &= (-\sin\theta, \cos\theta) \cdot \mathbf{P}_i + \mathbf{T}_y \end{aligned} \quad (2)$$

l_i can be eliminated yielding:

$$(\cos\theta, \sin\theta)\cdot\mathbf{P}_i + \mathbf{T}_x = ((-\sin\theta, \cos\theta)\cdot\mathbf{P}_i + \mathbf{T}_y)\cot\beta_i \quad (3)$$

This is a homogeneous linear equation in four unknowns

$$(P_{ix} - P_{iy}\cot\beta_i, P_{iy} + P_{ix}\cot\beta_i, 1, -\cot\beta_i)\cdot\begin{pmatrix}\cos\theta \\ \sin\theta \\ \mathbf{T}_x \\ \mathbf{T}_y\end{pmatrix} = 0.$$

We will denote by A_i the coefficient vectors which are computed from P_i and $\cot\beta_i$ and by W the vector of the unknown parameters. Taking equations for three such points yields the following equation.

$$AW = \begin{pmatrix} A_1^T \\ A_2^T \\ A_3^T \end{pmatrix} W = 0$$

In order to find W we apply SVD to A [14](chapter 2.6). This decomposes A into

$$A = USV^T$$

where U and V are orthogonal matrices and S is a diagonal matrix whose diagonal elements are non-negative numbers sorted by size. The last (fourth) vector of matrix V which corresponds to the smallest element in the diagonal of S is the solution to the problem.

As this is an homogeneous set of equations the solution for W is determined up-to a scale factor. This scale factor can be determined by exploiting the fact that W_1 and W_2 are $\cos\theta$ and $\sin\theta$ respectively and should therefore satisfy $W_1^2 + W_2^2 = 1$. Thus W is recovered. However also $-W$ could be the solution. To verify which of them is the correct solution we substitute the solution into the initial set of equations (2) and check if the right hand sides of the equations have the same sign as the $\cos\beta_i$ and $\sin\beta_i$ whose sign is known but was lost in the derivation step from (2) to (3). Only one of the W's will satisfy these constraints.

If there are additional bearing measurements each of them will add an additional row to A. Then the solution mentioned above will yield the optimal least squares solution to this linear system. i.e.

$$\arg\min_W ||AW||^2 \quad \text{subject to} \quad ||W||^2 = 1$$

3 Improved Linear Systems

Using SVD to solve our problem is very appealing because it is very efficient and has a linear computational complexity in the number of landmarks. The problem is that the solution obtained by SVD is not the optimal solution for the original problem we were trying to solve. The optimal solution we are seeking is the position $\mathbf{P}$ and rotation angle θ such that the bearings measured from that position and orientation $\hat{\beta}_{i(\mathbf{P},\theta)}$ are closest to the β_i's we measured in the experiment. i.e.

$$\arg\min_{\mathbf{P},\theta} \sum_i (\beta_i - \hat{\beta}_{i(\mathbf{P},\theta)})^2 \qquad (4)$$

For our linear system to yield close to optimal results we should try to make it fulfill the following three requirements. The first requirement is that in a total least-squares problem (like the one we are solving) each entry in the matrix A should have the same variance [9](Chapter 12.3). This variance is due to measurement errors. The second requirement is that

$$|A_i^T W|^2 = |\beta_i - \hat{\beta}_{i(\mathbf{P},\theta)}|^2 \qquad (5)$$

Thus the solution will satisfy (4). I.e. the error in each measurement β_i will have the same effect on the solution. If we know that certain measurements are more accurate than others this equation can be modified to account for this. The last requirement is that each of the entries of the matrix A be independent of the others. This requirement can not be fulfilled for entries in the same row of the matrix as they are all functions of a certain β_i but it is fulfilled for entries from different rows as we assume that the measurements are independent.

We will first deal with the first requirement. Looking at the definition of A_i it is obvious that this requirement is not fulfilled. The variance of each element $A_{i,j}$ is approximately $(\frac{\partial A_{i,j}}{\partial \beta_i}\sigma)^2$ where σ^2 is the variance of the bearing measurements (assuming for now that the variance for all measurements are equal). Thus looking at

$$\frac{\partial A_i}{\partial \beta_i} = \left(P_{iy}(\cot\beta_i^2 + 1), -P_{ix}(\cot\beta_i^2 + 1), 0, (\cot\beta_i^2 + 1)\right)$$

We can see that the variance values are very non-uniform, both within a row of A and between the rows. The third element has variance 0 and the others are multiples of $-(\cot\beta_i^2 + 1)$ which can be unlimited in magnitude. Thus a small measurement error can cause errors in different magnitudes for β_i's of different values.

The first step is to make the variance bounded. We will multiply each row by $\sin\beta_i$ yielding:

$$A_i = (P_{ix}\sin\beta_i - P_{iy}\cos\beta_i, P_{iy}\sin\beta_i + P_{ix}\cos\beta_i, \sin\beta_i, -\cos\beta_i) \qquad (6)$$

This makes the variance bounded.

$$\frac{\partial A_i}{\partial \beta_i} = (P_{iy}\sin\beta_i + P_{ix}\cos\beta_i, P_{iy}\cos\beta_i - P_{ix}\sin\beta_i, \cos\beta_i, \sin\beta_i)$$

The variance is bounded by $|P_i|^2\sigma^2$ for the first two entries but by σ^2 for the second two entries. To make the first two entries have similar values to the other two we apply a transformation to the world coordinates of the landmark positions $\mathbf{P}_i$. Once the solution is found the inverse transformation will be applied to it yielding the position in the original world coordinate system. This transformation is very similar to the one used in [12] and for the same reason which is to improve the quality of the result obtained from solving a linear system. The problem addressed in [12] was of course a different one.

Thus we compute the centroid $\mathbf{C}$ of the $\mathbf{P}_i$'s , and $SZ = \max_i |\mathbf{P}_i - \mathbf{C}|$ and apply the following transformation to the $\mathbf{P}_i$'s.

$$\mathbf{P}'_i = (\mathbf{P}_i - \mathbf{C})/SZ$$

Now the variances of all the entries have similar magnitudes. Applying these transformations to the original set of equations has a negligible computational cost but improve the quality of the estimates considerably.

The last step in our algorithm is to ensure that the (5) is enforced. This constraint is equivalent to:

$$\frac{\partial A_i W}{\partial \beta_i} = 1$$

However, W is unknown. Therefore we run SVD on the matrix A we just obtained and use the W obtained to compute

$$d_i = \frac{\partial A_i W}{\partial \beta_i}$$

We then multiply each row A_i by $1/d_i$ and re-apply the SVD on the new weighted version of A. This operation is repeated twice or three times until the process converges. Thus at a cost twice or three times as much as the original computation another improvement is achieved.

3.1 Experimental Results

To test our algorithms we have implemented them using Matlab. To demonstrate the improvement achieved by the transformation steps we ran an experiment in which several hundred sets of measurements with random noise were given to the different variants of the algorithm and a non-linear optimization algorithm. The difference in the measurement sets were only due to the random noise. The obtained robot positions computed by the different algorithms are presented in Figure 3 drawn to the same scale. As the difference in the quality of the different algorithms is so significant we had to plot each of the results at three different scales so the results of the different algorithms can be appreciated. The reduction in scattering of the results demonstrate the quality of the last two algorithms. The results are also presented in Table 1 showing the mean and variance of the results of the different algorithms and their running times. The difference in variance between the first and last algorithm is a factor of 5400 (factor of the standard deviations is 73.6)!!! Our final algorithm achieves results which are nearly identical to those of the non-linear optimization technique but at 1% of the running time! The accuracy of the estimates (the covariance matrix) can be also estimated directly by estimating the derivatives of the estimating function with respect to the input and output parameters. The details of this method are presented in [11, 1].

Now that we have chosen an algorithm we would like to show some results of running it on several environments. In each environment the landmarks have been positioned in certain positions and we measure in each position in the workspace the accuracy of the estimate by repeating the experiment several dozen times and assessing the accuracy as the square root of the determinant of the covariance matrix or its trace. The values are displayed on a log scale. In the first example shown in Fig. 4 the landmarks are arranged in two circles of the same radius. One with ten landmarks and the other with five. The most accurate results are achieved near the landmarks. The results become less accurate as we approach the circle's centers. When we move away from the circle the accuracy again is reduced. Obviously the circle with more landmarks yields better results. Had there been only one circle in the workspace, close to the circle very inaccurate results would have been obtained due to the inherent ambiguity mentioned above.

In Figure 5 we organized the landmarks in two perpendicular lines. Here again the results are very accurate near the landmarks. However on the lines between the landmarks, the accuracy is very low. This is due to the difficulty to determine the coordinate of the robot's position in the direction of the line.

In the last example shown in Figure 6 we positioned the landmarks randomly. The best results are obtained close to two landmarks which were positioned very close to each other and another accurate position was determined near a cluster of four landmarks.

4 Applications and Future Directions

In conclusion, we have presented a new efficient localization algorithm for a mobile robot from noisy bearing measurements. We presented our algebraic solution and explained why several transformations should be applied to the linear system of equations in order to improve the accuracy of the solution and showed experimentally that these transformations indeed improve the accuracy yielding results which are very close to the optimal results which usually re-

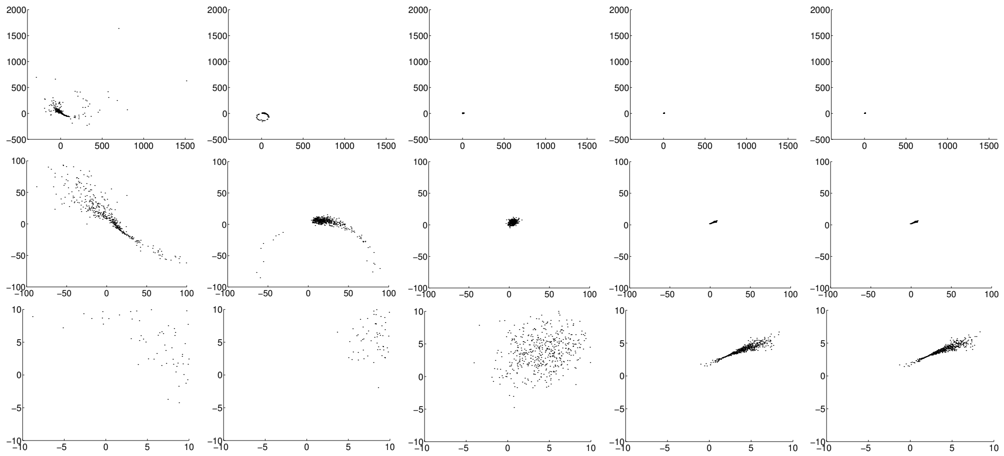

Figure 3: For each of the five algorithms we present the distribution of the results obtained from running it on 500 experiments. The correct answer is $(4,4)$ the landmarks lie within a circle of radius 100, and the measurement error is normally distributed with standard deviation of 5°. The results are plotted three times in different scales.

Method	Mean Position	Variance	Running Time
Initial Algorithm	(18.04,35.108)	(14733,12844)	0.0084476
Bounded Variance	(20.594,-1.7789)	(355,639.2)	0.0065014
Scaled data	(4.3223,3.8965)	(7.6949,6.6746)	0.0070153
Iterative weighted rows	(3.9996,3.9624)	(2.7276,0.70346)	0.011813
Non-Linear Optimization	(3.9307,3.9347)	(2.7204,0.7094)	**1.4277**

Table 1: A table showing the statistics of the experiment. For each algorithm the table shows the mean position obtained by the algorithm (should be (4,4)), the variance in the x and y coordinates and the average running time in seconds for running each experiment. As can be seen the quality of the results improve from one algorithm to the next. The fourth algorithm gets approximately the same results as the non-linear optimization algorithm but at 1% of the computational cost!!!. Thus it should be used instead of the non-linear optimization.

quire non-linear optimization techniques. Our final algorithm is also very efficient running more than a hundred times faster than the non-linear optimization technique. We therefore recommend that our algorithm be used for robot localization from landmark bearing measurements.

The analysis presented here can be used to develop more accurate algorithms and study others. For example in studying the algorithm to solve problem in computer vision of estimating the fundamental matrix [12]. We can see that the scaling of the input that was performed there causes the variances between the different entries in the coefficient matrix to be more similar improving the quality of the results. However as one of the columns in the matrix is a constant 1 (variance 0), not all entries in the matrix have similar variances and thus the solution that this algorithm achieves is not as close to the optimal solution as we achieved. This is obviously due to the difference between the problems solved.

The main use of this algorithm is to accurately recover the position and orientation of the robot at very low computational cost. As shown above we can also estimate the accuracy of our estimate over the entire workspace. This can be used to compute optimal motion plans which take into account not only the distance to the goal but also the number of times robot's position must be estimated. This number is determined by the accuracy of each estimate (from the value we have computed at the position the measurement was made) and how accurately does the robot need to know its location (less accurate when its far from an obstacle and more accurate when its near an obstacle or close to a goal position that has to be reached at a certain precision). This problem is discussed in detail in [2, 3].

The inverse problem is also interesting. In this case requirements are made on the accuracy of the robot's position at certain places and on certain paths in the workspace. The task is to determine where to place

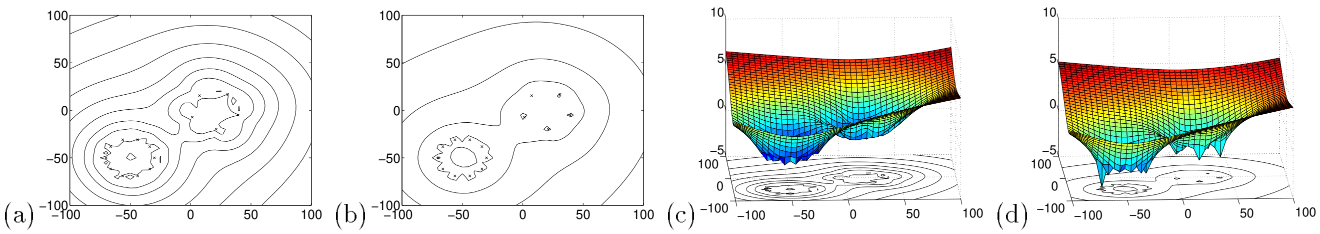

Figure 4: A scene with two circles of landmarks with 10 and 5 landmarks each, landmarks are marked with x's. (a) A contour map of the log of the trace of the covariance map; (b) A contour map of the log of the determinant of the covariance map; (c) & (d) the surfaces of these two uncertainty functions. Best results obtained near the landmarks of the circle with more landmarks.

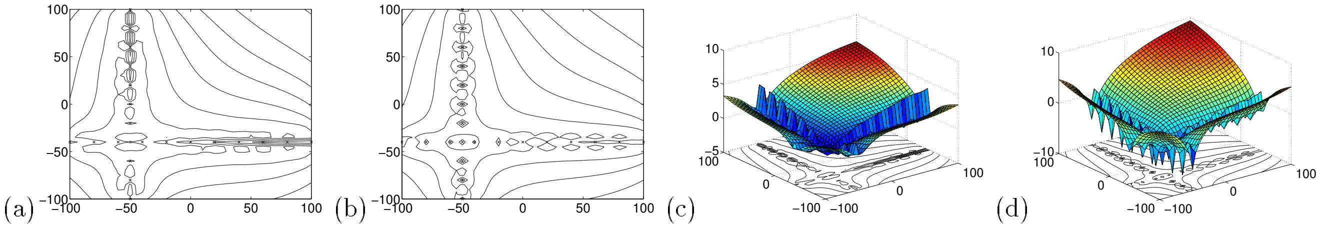

Figure 5: A scene with two perpendicular lines of landmarks: (a) A contour map of the log of the trace of the covariance map; (b) A contour map of the log of the determinant of the covariance map; (c) & (d) the surfaces of these two uncertainty functions. The localization algorithm has problem localizing the robot on the lines between the landmarks. The exact position of the robot on the line is hard to determine.

the landmarks in the workspace. This problem has been investigated in [16]. Our method can be used as the basic building block in research on these types of problems.

Acknowledgments

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References

[1] A. Adam, E. Rivlin, and I. Shimshoni. Computing the sensory uncertainty field of a vision-based localization sensor. In *IEEE Int. Conf. on Robotics and Automation*, pages 2993–2999, 2000.

[2] A. Adam, E. Rivlin, and I. Shimshoni. On designing meta motion planners A: Model and framework. In *IEEE Int. Conf. on Robotics and Automation*, 2001.

[3] A. Adam, E. Rivlin, and I. Shimshoni. On designing meta motion planners B: Algorithm and applications. In *IEEE Int. Conf. on Robotics and Automation*, 2001.

[4] K. Aström and M. Oskarsson. Solutions and ambiguities of the structure and motion problem for 1D retinal views. In *Proc. Scandinavian Conf. Image Analysis*, 1999.

[5] M. Betke and L. Gurvits. Mobile robot localization using landmarks. *IEEE Trans. on Robotics and Automation*, 13(2):251–263, 1997.

[6] T. Celinski and B. McCarragher. Achieving efficient data fusion through integration of sensory perception control and sensor fusion. In *IEEE Int. Conf. on Robotics and Automation*, pages 1960–1965, 1999.

[7] T. Fraichard and R. Mermond. Path planning with uncertainty for car-like robots. In *IEEE Int. Conf. on Robotics and Automation*, pages 27–32, 1998.

[8] A. Garulli and A. Vicino. Set membership localization of mobile robots via angle measurements. *IEEE Trans. on Robotics and Automation*, 2000.

[9] G.H. Golub and C.F. Van-Loan. *Matrix Computations*. The Johns Hopkins University Press, Baltimore, 1996.

[10] U.D. Hanebeck and G. Schmidt. Set theoretic localization of fast mobile robots using angle measurements. In *IEEE Int. Conf. on Robotics and Automation*, pages 1387–1394, 1996.

[11] R.M. Haralick. Propogating covariance in computer vision. *Int. J. of Patt. Recog. and Art. Intell.*, 10:561–572, 1996.

[12] R.I. Hartley. In defense of the eight-point algorithm. *IEEE Trans. Patt. Anal. Mach. Intell.*, 19(6):580–593, June 1997.

[13] A. Lambert and N. L. Fort-Piat. Safe actions and observation planning for mobile robots. In *IEEE Int. Conf. on Robotics and Automation*, pages 1341–1346, 1999.

[14] W. Press, B. Flannery, S. Teukolsky, and W. Vetterling. *Numerical Recipes in C*. Cambridge University Press, 1988.

[15] N. Roy, W. Bugard, D. Fox, and S. Thrun. Coastal navigation - mobile robot navigation with uncertainty in dynamic environments. In *IEEE Int. Conf. on Robotics and Automation*, pages 35–40, 1999.

[16] D. Sinriech and S. Shoval. Landmark configuration for absolute positioning of autonomous vehicles. *IIE Transactions*, 32:613–624, 2000.

[17] K.T. Sutherland and W.B. Thompson. Localizing in unstructured environments: dealing with the errors. *IEEE Trans. on Robotics and Automation*, 10(6):740–754, 1994.

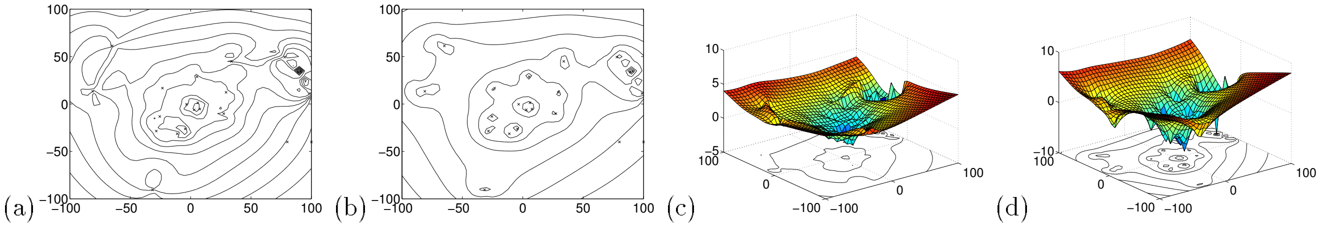

Figure 6: A scene with a set of randomly placed landmarks: (a) A contour map of the log of the trace of the covariance map; (b) A contour map of the log of the determinant of the covariance map; (c) & (d) the surfaces of these two uncertainty functions. The localization algorithm does best close to two very close landmarks and does also well next to four landmarks a bit further apart.

[18] H. Takeda, C. Facchinetti, and J.C. Latombe. Planning the motions of a mobile robot in a sensory uncertainty field. *IEEE Trans. Patt. Anal. Mach. Intell.*, 16(10):1002–1017, October 1994.

[19] U. Wiklund, U. Anderson, and K. Hyyppa. AGV navigation by angle measurements. In *Proc. of 6th Inter. Conf. on AGV Systems*, pages 199–212, 1988.