Exercise 2 – 048972

1. **Two Homographies**
   Assume there are two images. In those images two planar objects are visible. The homographies of the images of the two planes in the two images are A and B. Which points are the fixed points of AB^{-1} i.e. points which satisfy $p \cong AB^{-1}p$. Which points in the 3D world project to those points. How can you compute their image from AB^{-1}.

2. **Magic Triangle**
   Legend says that if you measured three vanishing points for three orthogonal direction v_x,v_y and v_z then if you make a triangle from them the place where all lines which start at corners of the triangle and are orthogonal to the opposite edge of the triangle is the principal point (u_0, v_0).

   (a) Try that on image L10.jpg or abbey1b.jpg.

   (b) Given two lines l_1 and l_2 what equation can you write which is equivalent to the fact that they are orthogonal.

   (c) Prove that if the points are from a normalized image (i.e. the plane z=1) the lines intersect at the origin. i.e. the line $v_y - v_z$ is orthogonal to $v_x - (0, 0, 1)$.

   (d) How do you convert v in normalized coordinates to a position in the image when given the matrix K.

   (e) Assuming that when $f_x = f_y$ and that there is no skew (i.e. $K(1, 2) = 0$) show that the lines intersect at (u_0, v_0).

   (f) What can you do if you know that $f_x = 1.5 * f_y$.

   (g) Does the magic triangle trick work for a general K matrix.

3. **Fundamental Matrices and Planes**
   Consider two images I and I' from which pairs of corresponding points p_i and p'_i are given. In the scene there exists a plane π and an homography H_π such that when p_i lies on π $p'_i \cong H_\pi p_i$. For all points p_i let $p''_i \cong H_\pi p_i$. What is the special structure of the fundamental matrix relating the p'_i's and the p''_i's and the corresponding epipolar lines and epipoles.

4. **Planar Mirrors**
   Consider an image which includes in it a planar mirror. For simplicity assume that the mirror lies in the plane $Z = 0$. The camera position is O and the camera orientation with respect to the world coordinate frame is R.

   Each real point appearing in the image has its reflection also appearing in the image. Therefore we can consider this image as an image plane.

   - Give an equation describing the fundamental matrix F as a function of the internal calibration matrix K and the other parameters mentioned above.
   - How many point matches are required in order to compute F.
   - Take a picture of this type and compute for it the fundamental matrix.