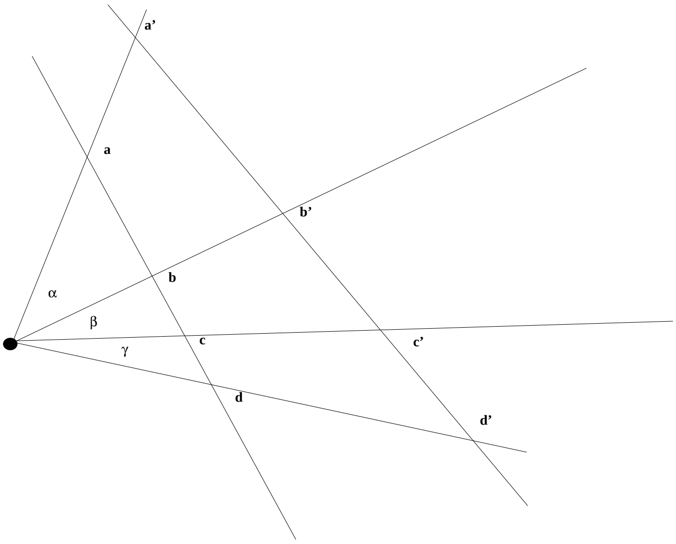
Exercise 1 – 048972

1. **SVD**

   Given a $M \times N$ matrix A where $M \gg N$ (e.g $M = 100000, N = 100$). Computing SVD(A) is time consuming. Number of flops is $4M^2N + 8MN^2 + 9N^3$. Therefore:

   (a) If $A = USV^T$ what is the SVD of AA^T and A^TA.

   (b) If you have performed $SVD(AA^T)$ or $SVD(A^TA)$ how can you compute the $SVD(A)$.



2. **Cross-ratio**

   (a) The cross-ratio of four points on a line is defined as:

   $$cr(a, b, c, d) = \frac{d(a, b)d(c, d)}{d(a, c)d(b, d)}$$

   where $d(a, b)$ is the distance from a to b. Prove that the cross-ratio value is the same for (a, b, c, d) and (a', b', c', d'). Show that the cross-ratio can be expressed as a function of the angles α, β, γ.

hint: use the theorem of sines. That is, given a triangle with edges length a, b, c and opposite angles α, β, γ then

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

(b) What is the cross-ratio value if $d = \infty$ as a function of the distances between a, b, c.

(c) Why do two images of a line with four points marked on it have the same cross-ratio?

3. Take the enclosed image (abbey1b.jpg) and perform the following steps in order to recover the three vanishing points in the image.

   (a) Compute the equations of the lines of the crosswalk from several points on each line. Find the points and using matlab compute the equations of the lines.

   (b) Find the intersection point of the lines (vanishing point) using matlab. i.e. find the point closest to all the lines in a least-squares sense.

   (c) Find the other vanishing point (orthogonal to the crosswalk lines) using the cross-ratio.

   (d) Compute the equation of the vanishing line.

   (e) Is there data in the image to estimate the vertical direction vanishing point? If so estimate it also.

   (f) Compute Vanishing points for the second image too (L10.jpg).

4. We know that the method described above for computing the vanishing point is wrong. What is really needed is a non-linear optimization method. A component of the method is when you are given a candidate vanishing point vp for each line take the points from which the line was computed and find the line which goes through vp which is closest in a least squares sense to the set of points. How should you modify the method we discussed in class for estimating a line from points to fit this case.

5. In class we studied Tsai's calibration algorithm (the one which is able to solve part of the calibration parameters even though radial distortion exists).

   Consider the following problem. You are given an image of a chessboard taken from some unknown position. Suggest a variant of Tsai's algorithm for recovering the Homography between the image of the chessboard and the chessboard which takes into account radial distortion. There is no need to implement the algorithm. Just explain how it works.

6. (a) Given a unit vector $\mathbf{v}$ in 3D which is a point on the unit sphere. What equation do all the unit vectors $\mathbf{u}$ whose angles with $\mathbf{v}$ is $90°$ satisfy.

   (b) What does this set of points look like geometrically (point, line circle, conic...).

   (c) Assuming that the image of direction $\mathbf{v}$ (at infinity) is p. Using the image of the absolute conic (IAC) ω, What do the images q of the unit vectors $\mathbf{u}$ whose angles with $\mathbf{v}$ is $90°$ satisfy.

   (d) What does this set of points on the image look like geometrically (point, line circle, conic...).

   (e) Solve the previous two items for the general case where the angle between the vectors is θ.

   (f) Using the fact that $\omega = K^{-T}K^{-1}$, where K can be any matrix, what vectors p satisfy $p^T\omega p = 0$.

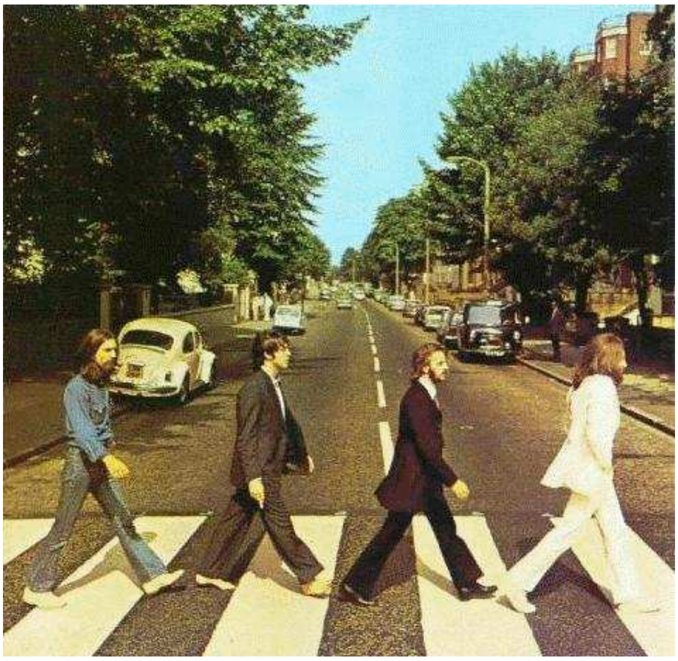


Figure 1: Abbey Road abbey1b.jpg

Figure 2: Living Room scene L10